Micro C – Spring 2013 – Re-Exam Solutions

1.

(a) There are two equilibria in pure strategies: (A, L) and (C, R). To find the equilibrium in mixed strategies, we can exclude strategy B, since it is strictly dominated by strategy A, and strictly dominated strategies are never played with positive probability in any equilibrium.

Let p be the probability that 1 plays A, and q the probability that 2 plays L. In a mixed-strategy NE, the players need to be indifferent between their strategies, so that for player 2:

$$\begin{aligned} EU(L) &= EU(R) \\ 3p + 1 - p &= 1 - p \\ 3p &= 0 \\ p &= 0 \end{aligned}$$

[One can also simply argue that p has to be zero because L weakly dominates R.] Player 1 is indifferent between A and C if:

$$\begin{aligned} EU(A) &= EU(C) \\ 3q + 1 - q &= 2 - 2q \\ 4q &= 1 \\ q &= \frac{1}{4} \end{aligned}$$

If $q < \frac{1}{4}$, player 1 still prefers C. The set of mixed strategy equilibria is hence $((0,1), (q^*, 1-q^*))$, where $q^* \leq \frac{1}{4}$. (Or (p^*, q^*) with $p = 0$ and $q^* \leq \frac{1}{4}$.)

In the mixed-strategy equilibria, player 1 has the expected payoff $2 - 2q^*$, which is between $\frac{3}{2}$ and 2. Therefore, 1 has the highest expected payoff in the pure-strategy Nash Equilibrium (A, L). [A complete answer requires finding all the mixed-strategy NE and calculating the expected payoff for player 1 from the mixed-strategy NE. If the student only finds the mixed-strategy NE where

$q^* = \frac{1}{4}$, however, and then continues correctly, there is only a minor reduction in points because the student has shown the ability to apply the main technique for finding mixed-strategy NE.]

(b) In the second round, the players always need to play a Nash Equilibrium. But they can play different Nash Equilibria, depending in what happens in the first round. If they play (A, L) in the second round if the outcome in the first round was (B, L) and (C, R) otherwise, the payoffs in the first round become:

Player 2

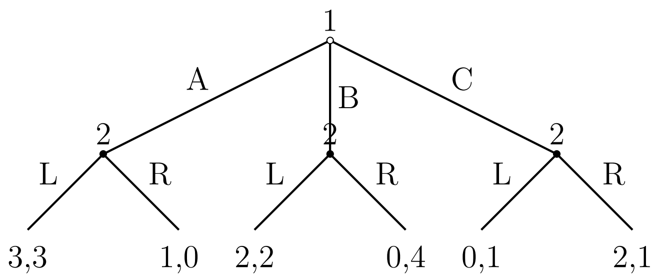
		L	R
	A	$5, 4$	$3, 1$
Player 1	B	$5, 5$	$2, 5$
	C	$2, 2$	$4, 2$

It is now an equilibrium for the players to play (B, L) in the first round. The corresponding SPNE is

$$(BCCACCC,\ LRRLRRR)$$

where the first action is the action in the first stage, the second action is the action after (A, L), the third action the action chosen after (A, R) and so on. [An informal description is also sufficient, as long as it is complete and correct.]

(c) The game looks like this:



(d) In G: $S_1 = \{A, B, C\}$ and $S_2 = \{L, R\}$

In the extensive-form game from (c): $S_1 = \{A, B, C\}$ and

$$S_2 = \{LLL, LLR, LRL, RLL, LRR, RLR, RRL, RRR\}$$

[The sequence of strategies doesn't matter, since strategy sets are unordered.]

The strategy set of player 2 in the game from (c) is much larger, since player 2 now can react to player 1. We therefore need to specify a reaction by player 2 to every possible choice that player 1 can make.

(e) There are two SPNE: (A, LRL) and (A, LRR).

2. A is the PBE, B is the SPNE and C the NE.

3.

(a) The firm's profit functions are

$$\begin{aligned}\pi_1(q_1, q_2) &= (36 - q_1 - q_2 - c)q_1 \\ \pi_2(q_1, q_2) &= (36 - q_1 - q_2 - c)q_2\end{aligned}$$

The first-order conditions give us the best-response functions

$$\begin{aligned}q_1 &= \frac{36 - q_2 - c}{2} \\ q_2 &= \frac{36 - q_1 - c}{2}\end{aligned}$$

We use this system of equations to get

$$q_1^* = \frac{36 - c}{3} = q_2^*.$$

(b) If the two firms merge, there is only one firm that acts as a monopolist. Its profit is $\pi(q) = (36-q-c)q$, and the first-order condition gives us the monopoly quantity

$$q^M = \frac{36 - c}{2}.$$

Profit in the monopoly is $\frac{(36-c)^2}{4}$. In the Nash Equilibrium, each firm has a profit of $\frac{(36-c)^2}{9}$, so that total profit in the Nash Equilibrium is $\frac{2}{9}(36-c)^2 < \frac{1}{4}(36-c)^2$.

The profit is higher in monopoly since the monopolist can keep the quantity low and keep prices at a high level. This is not sustainable with two firms, since each firm has an incentive to produce more than half the monopoly quantity, so that profits in the Nash Equilibrium with two firms are lower. One can also say that setting a higher q_i benefits firm i, but has a negative externality on the profit of firm j ($j \neq i$).

(c) Type spaces: $T_1 = \{1\}$ and $T_2 = \{H, L\}$ (it is not necessary to give the type space of firm 1)

Action spaces: $A_1 = [0, \infty)$ and $A_2 = [0, \infty)$

Strategy spaces: $S_1 = [0, \infty)$ and $S_2[0, \infty) \times [0, \infty) = [0, \infty)^2$

Beliefs: $\mu_1(t_2 = H) = \theta$, $\mu_1(t_2 = L) = 1 - \theta$

(d) The best-response functions of firm 2 are now

$$\begin{aligned} q_2^H &= \frac{36 - q_1 - c_H}{2} \\ q_2^L &= \frac{36 - q_1 - c_L}{2} \end{aligned}$$

while the best-reponse function of firm 1 is

$$q_1 = \frac{36 - \theta q_2^H - (1-\theta) q_2^L - c}{2}.$$

From solving this system of three equations we get

$$\begin{aligned} q_1 &= \frac{36 + \theta c_H + (1-\theta) c_L - 2c}{3} \\ q_2^H &= \frac{72 - \theta c_H - (4-\theta) c_L + 2c}{6} \\ q_2^L &= \frac{72 - (3+\theta) c_H - (1-\theta) c_L + 2c}{6} \end{aligned}$$

If we set $c = c_H = c_L$, we get

$$\begin{aligned} q_1 &= \frac{36 + \theta c + (1-\theta) c - 2c}{3} = \frac{36 - c}{3} \\ q_2^H &= \frac{72 - \theta c - (4-\theta) c + 2c}{6} = \frac{36 - c}{3} \\ q_2^L &= \frac{72 - (3+\theta) c - (1-\theta) c + 2c}{6} = \frac{36 - c}{3} \end{aligned}$$

4.

(a) There is one separating equilibrium, $((R, L), (d, u), p = 0, q = 1)$, and two pooling PBE: $\left((L, L), (u, d), p = \frac{1}{2}, q \leq \frac{1}{4}\right)$ and $\left((R, R), (d, u), p \leq \frac{1}{5}, q = \frac{1}{2}\right)$.

(b) In the PBE $\left((L, L), (u, d), p = \frac{1}{2}, q \leq \frac{1}{4}\right)$, R is equilibrium dominated for t_1 and signaling requirement 6 would therefore require $q = 1$. But that is not possible if $q \leq \frac{1}{4}$. Only the PBE $\left((R, R), (d, u), p = 0, q = \frac{1}{2}\right)$ fulfills signaling requirement 6, since L is equilibrium dominated for t_1. Since every PBE that fulfills requirement 6 also fulfills signaling requirement 5, this PBE also fulfills signaling requirement 5.